TWO-WAY TABLES — WALKING AND BICYCLING TO WORK

TEACHER VERSION

Subject Level:
Middle School Math

Grade Level:
8

Approx. Time Required:
60–120 minutes

Learning Objectives:

- Students will be able to interpret two-way tables containing data on two categorical variables collected from the same subjects.
- Students will be able to describe possible associations between two categorical variables by calculating relative frequencies and conditional relative frequencies.

Activity Description

Students will use data from the 2019 American Community Survey (ACS) to compare how men and women in two cities used nonmotorized transportation to get to work. Students will calculate relative frequencies and conditional relative frequencies from data in two-way tables.

Suggested Grade Level:
8

Approximate Time Required:
60–120 minutes

Learning Objectives:

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Topics:

- Two-way tables

Skills Taught:

- Reading and interpreting data in two-way tables
- Calculating relative frequencies and conditional relative frequencies

Materials Required

- The student version of this activity, 8 pages
- Calculators

Data in this activity come from the 2019 ACS. The ACS, conducted monthly by the U.S. Census Bureau, is designed to show how communities are changing. Through asking questions of a sample of the population, it produces national data on more than 35 categories of information, such as education, income, housing, and employment.

If desired, teachers can view the source for data used in this activity by clicking on this link:
SEX OF WORKERS BY MEANS OF TRANSPORTATION TO WORK 2019 ACS, 1-Year Estimates.

For more information to help you introduce your students to the Census Bureau, read "*Census Bureau 101 for Students.*" This information sheet can be printed and passed out to your students as well.

Standards Addressed

See charts below. For more information, read
"*Overview of Education Standards and Guidelines Addressed in Statistics in Schools Activities.*"

Common Core State Standards for Mathematics

Standard	Domain	Cluster
CCSS.MATH.CONTENT.8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	8 SP – Statistics & Probability	Investigate patterns of association in bivariate data.

Common Core State Standards for Mathematical Practice

Standard
CCSS.MATH.PRACTICE.MP4 Model with mathematics. Students will use the frequencies provided to calculate the relative frequencies and conditional relative frequencies and describe a possible association between the variables.
CCSS.MATH.PRACTICE.MP6 Attend to precision. Students will accurately calculate relative frequencies and conditional frequencies.

National Council of Teachers of Mathematics' Principles and Standards for School Mathematics

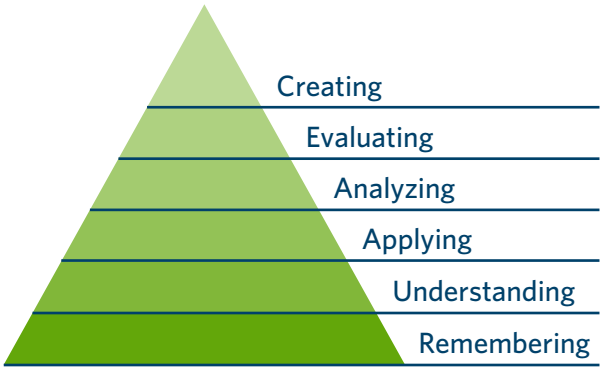
Content Standard	Students should be able to:	Expectation for Grade Band
Data Analysis and Probability	Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.	Formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population. Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.
Data Analysis and Probability	Select and use appropriate statistical methods to analyze data.	Find, use, and interpret measures of center and spread, including mean and interquartile range. Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots.
Data Analysis and Probability	Develop and evaluate inferences and predictions that are based on data.	Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken.

Guidelines for Assessment and Instruction in Statistics Education

GAISE	Level A	Level B	Level C
Formulate Questions		X	
Collect Data			
Analyze Data		X	
Interpret Results		X	

Bloom's Taxonomy

Students will ***analyze*** two-way tables containing data on two categorical variables.



Teacher Notes

Before the Activity

Students must understand the following key terms:

- **Categorical variable** – a variable that is not numerical, such as a name or label, that places an object into one of several groups or categories. (e.g., the color of a ball or the breed of a dog)
- **Conditional relative frequency** – the ratio of a joint relative frequency and related marginal relative frequency
- **Frequency** – the number of times an object item, number, or event occurs in a data set
- **Joint relative frequency** – a relative frequency calculated by dividing a cell frequency in a two-way table by the total number of observations in the table
- **Marginal relative frequency** – a relative frequency calculated by dividing a row total by the total number of observations in the table
- **Quantitative variable** – a variable that is numerical, meaning it represents a measurable quantity (e.g., the population size of a city)
- **Relative frequency** – the frequency of a particular category or cell divided by the total number of observations
- **Two-way table** – a table containing data in two rows and two columns (also called a contingency table) used to examine relationships between categorical variables.

Students should have a basic understanding of the following ideas and concepts:

- How to convert decimals to percentages
- How to interpret two-way tables

Teachers will ask students how people in their household get to work (if those people work outside the home). Teachers will lead a class discussion about why people use particular modes of transportation: cost, convenience, availability, environmental factors, commuting time or distance, etc.

Teachers will help students understand a conditional relative frequency, perhaps by drawing stick figure diagrams to illustrate the concept. For example, teachers could draw 20 stick figures to represent the number of men in a particular city, showing that 19 of them (95 percent) walked to work and one of them (5 percent) biked. Teachers could then draw another 20 stick figures to represent the number of workers who biked to work in that city, showing that 18 of them (90 percent) were men and two of them (10 percent) were women.

Teachers should note that the data used are estimates and have a margin of error. A margin of error is the difference between an estimate and its upper or lower confidence bounds.

During the Activity

Teachers should note that in question 1 of part 1, the numbers were calculated by working backward from the percentages in the source data.

Teachers will make sure that students are calculating the relative frequencies correctly.

If teachers find that this activity is too difficult for students, they could consider simplifying it by:

- Skipping question 8 in part 4 of the activity, where students write and answer their own questions
- Using data for only one city

If teachers find that this activity should be more visual, they could create bar graphs illustrating the data.

After the Activity

Teachers will check for student understanding, asking students to share the questions they wrote and their answers for them.

Teachers will direct students to share their news articles from part 5 with the class.

Extension Ideas

- Teachers could ask students to make calculations using data for other cities or modes of transportation (which can be gathered from the source data).
- Teachers could adapt this activity for other GAISE levels (read "*Overview of Education Standards and Guidelines Addressed in Statistics in Schools Activities*" for more information):
  - Level A – Conduct a survey of how people in each student's household get to work, determine the two most popular modes of transportation, and have students create and interpret a two-way table that compares the men and women and their preferred mode of transportation.
  - Level C – Have students use the source data and formulate their own questions about commuting patterns, and then tabulate, analyze, and interpret the data selected.
- For part 5 of the activity, teachers could direct students to write an opinion piece instead of a news article. Teachers could also ask students to role-play, as "journalists" interviewing "commuters" from each city.

Student Activity

Click ***here*** to download a printable version for students.

Student Learning Objectives

- I will be able to interpret two-way tables containing data on two categorical variables collected from the same subjects.
- I will be able to describe possible associations between two categorical variables by calculating relative frequencies and conditional relative frequencies.

Part 1 – Make Data Observations

Cambridge, Massachusetts, and Columbia, South Carolina, are among the cities that had the highest rates of workers walking to work in 2019. Walking and biking are examples of nonmotorized transportation that the U.S. Census Bureau counts in its American Community Survey.

The ACS is conducted monthly by the Census Bureau and is designed to show how communities are changing. Through asking questions of a sample of the population, it produces national data on more than 35 categories of information, such as education, income, housing, and employment.

In this activity, you will use two-way tables to compare ACS data for three categorical variables:

- **City:** Cambridge and Columbia
- **Mode of transportation:** walking and biking
- **Gender:** men and women

1. Examine Tables 1 and 2 below and write down your observations and questions about the data. These data show workers aged 16 and older — in Cambridge and in Columbia — who walked or biked to work in 2019.

Table 1

Cambridge, Massachusetts	Walked	Biked	Total
Men	8,981	4,471	13,452
Women	7,809	1,753	9,562
Total	16,790	6,224	23,014

Table 2

Columbia, South Carolina	Walked	Biked	Total
Men	9,998	344	10,342
Women	6,155	118	6,273
Total	16,153	462	16,615

Student answers will vary but could include the following:

Few workers biked to work in Columbia.

The total number of workers who walked to work is about the same in each city.

Very few women in Columbia biked to work.

The data don't include workers who rode skateboards or scooters to work.

Were people who ran to work counted as people who walked to work?

Part 2 – Calculate Relative Frequencies

Calculate the relative frequencies for Tables 1 and 2 by dividing the value in each cell by the total for that table. (For example, to calculate the relative frequency of men in Cambridge who walked to work, divide 8,981 by 23,014. You should get a decimal between 0 and 1 that, after moving the decimal point to the right two places, represents the percentage of men in Cambridge who walked to work.) Round your numbers to the nearest whole percentage and place them in Tables 3 and 4.

Table 3

Cambridge, Massachusetts	Walked	Biked	Total
Men	**39%**	**19%**	**58%**
Women	**34%**	**8%**	**42%**
Total	**73%**	**27%**	100%

Table 4

Columbia, South Carolina	Walked	Biked	Total
Men	**60%**	**2%**	**62%**
Women	**37%**	**1%**	**38%**
Total	**97%**	**3%**	100%

1. Why is it that both numbers in the "Total" cells for Tables 3 and 4 equal 100 percent?

   The total is always 100 percent because the sum of all the relative frequencies in a table must equal one whole.

2. Which city had a greater number of workers who walked to work (of the workers who use nonmotorized transportation to get to work)?

   Cambridge (16,790 people vs. 16,153 people for Columbia)

3. Which city had a greater proportion of workers who walked to work (of the workers who use nonmotorized transportation to get to work)?

   Columbia (97 percent vs. 73 percent for Cambridge)

4. Do you see any differences in the proportions of men and women who bike or walk to work, out of those that use nonmotorized transportation to get to work? Explain, citing the data.

   Student answers will vary but could include the following: In Cambridge, walking to work was more popular than biking (73 percent vs. 27 percent), and more men out of all workers who use nonmotorized transportation to get to work, walked and biked to work than women (58 percent vs. 42 percent). Biking to work was not very popular in Columbia, as only 3 percent of these workers did so.

Part 3 – Calculate Conditional Relative Frequencies by Row

Sometimes it is necessary to examine only one row in a data set displayed in a two-way table. For example, to determine what percentage of women (using nonmotorized transportation) in Cambridge walked to work and what percentage biked, you could divide the value in each corresponding cell of Table 1 by the total for that row.

In Table 5 below, the relative frequencies by row for Cambridge have been calculated from Table 1 and written as percentages, each rounded to the nearest whole percentage. Using the data from Table 2, calculate the percentages for Columbia and record them in Table 6, following the same format.

Table 5

Cambridge, Massachusetts	Walked	Biked	Total
Men	67%	33%	100%
Women	82%	18%	100%
Men and Women	73%	27%	100%

Table 6

Columbia, South Carolina	Walked	Biked	Total
Men	**97%**	**3%**	**100%**
Women	**98%**	**2%**	**100%**
Men and Women	**97%**	**3%**	100%

1. Why are all the values in the "Total" column equal to 100 percent?

   This data set includes only workers who used nonmotorized transportation (walking and biking) to get to work. When you add the percentage of men who walked and the percentage of men who biked, for example, you get 100 percent of the men who used nonmotorized transportation.

2. Of the women in Cambridge who used nonmotorized transportation to get to work, what percentage walked and what percentage biked?

   82 percent walked and 18 percent biked.

3. Of the women in Columbia who used nonmotorized transportation to get to work, what percentage biked?

   2 percent biked.

4. Of the men in Cambridge who used nonmotorized transportation to get to work, what percentage biked?

   33 percent biked.

5. Of both the men and the women in Cambridge who used nonmotorized transportation to get to work, what percentage walked?

   73 percent walked.

Part 4 – Calculate Conditional Relative Frequencies by Column

Sometimes it is necessary to examine only one column from a data set. For example, to determine the percentage of workers who biked to work and who are women — as opposed to the percentage of all female nonmotorized commuters who biked to work — you could divide the value in the center cell of Tables 1 and 2 by the total for that column.

In Table 7 below, the relative frequencies by column for Cambridge have been calculated from Table 1 and written as percentages, each rounded to the nearest whole percentage. Using the data from Table 2, calculate the relative frequencies by column for Columbia and record them in Table 8, following the same format.

Table 7

Cambridge, Massachusetts	Walked	Biked	Total
Men	53%	72%	58%
Women	47%	28%	42%
Total	100%	100%	100%

Table 8

Columbia, South Carolina	Walked	Biked	Total
Men	**62%**	**74%**	**62%**
Women	**38%**	**26%**	**38%**
Total	**100%**	**100%**	100%

1. Why are all the values in the "Total" row equal to 100 percent?

   Men and women are the only two gender categories. When you add the percentage of men who walked to work and the percentage of women who walked to work, you get 100 percent of the workers who walked to work.

2. Of all workers in Cambridge who used nonmotorized transportation to get to work, what percentage of those who biked were women?

   28 percent

3. Of all workers in Cambridge who used nonmotorized transportation to get to work, what percentage of those who biked were men?

   72 percent

4. Calculate the difference in percentage points between the percentage of men and the percentage of women in Cambridge who biked to work:

   44 percentage points

5. Of the Columbia workers who used nonmotorized transportation, what percentage of those who biked were women?

   26 percent

6. Of the Columbia workers who used nonmotorized transportation, what percentage of those who biked were men?

   74 percent

7. Calculate the difference in percentage points between the percentage of men and the percentage of women in Columbia who biked to work:

   48 percentage points

8. What are some other questions you could answer with these data, using any of the two-way tables? Write three questions and answer them.

   Student questions and answers below will vary. Questions should be similar to those above (just using different variables).

   a.

   b.

   c.

Part 5 – Draw Conclusions

Write a three-paragraph news article — with a headline — that summarizes what you have learned. Think about one overarching idea that compares the two cities, the two genders, or the two modes of transportation.

In the first paragraph, write your overarching idea as a thesis statement, supporting it with specific data. Be sure to use appropriate mathematical language. In the second paragraph, explain why you think your thesis statement is true based on your general knowledge of these two cities, possible preferences of the genders, or the modes of transportation. In the third paragraph, restate your thesis and summarize your supporting points.

The content of students' news articles will vary. Possible headlines include:

- **More workers walk than bike to work**
- **Biking to work is not popular in Columbia, South Carolina**
- **Cambridge, Massachusetts is a good place to walk to work**